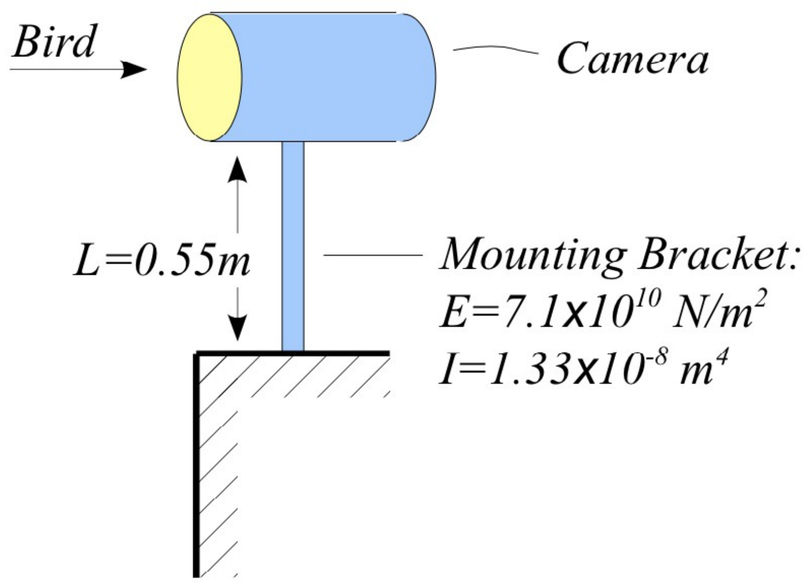
**Practice Problem Set 5C**
*Impulsive Forced Vibration*

Description:

- A 1-kg bird flies into a 3-kg security camera. If the bird is flying at 72 km/hr when it strikes the camera, and if the length of the mounting bracket ($L$) is 0.55 m, find the maximum deflection of the bracket caused by the impact, in meters.




- It's up to you to assume any level of damping.
- Sketch an $x$ vs $t$ graph along with your final answer.

# SCROLL

# DOWN

# FOR

# SOLUTION

***(But don't get tempted by the dark side. Resist! Use the, um, Force?)***

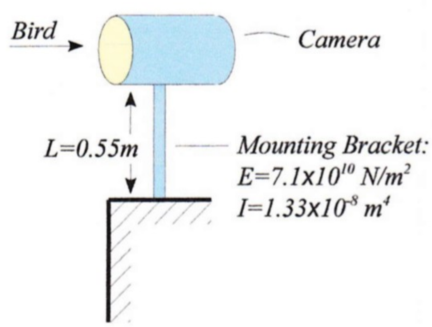
# ARE

# YOU

# SURE?

***(Go back up and think harder? Also, what exactly are you looking for in the solution below?)***

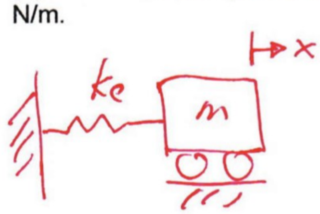
# SOLUTION




(a). State your assumptions.

- No damping
- Uniform mass (beam)
- Beam mass negligible
- Small deformation
- Frictionless
- Linear momentum is conserved between bird & camera

(b). Model (and sketch) this as a spring-mass system, and calculate the equivalent stiffness ($k_e$), in N/m.



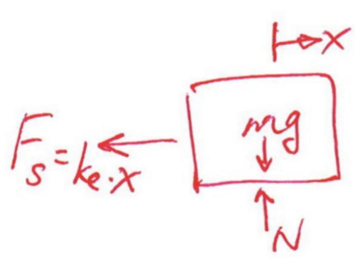

Bending:

$$k_e = \frac{3EI}{L^3} = \frac{3 \times 7.1 \times 10^{10} \times 1.33 \times 10^{-8}}{0.55^3}$$

$$= \boxed{17{,}070\ \frac{N}{m}}$$

(c). Draw the free-body and inertia diagrams. Fully label and define all necessary variables. [Hint: A system initially at rest and subject to an impulsive force is equivalent to *free* vibration with $x_o=0$ and $\dot{x}_o=\frac{I}{m}$ where $I$ is the impulse.]

FBD

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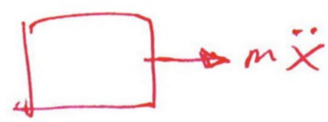


(d). Derive the differential equation of motion.

$$\Sigma F_x = m\ddot{x}$$

$$\Rightarrow -F_s = m\ddot{x}$$

$$\Rightarrow -k_e \cdot x = m\ddot{x}$$

$$\Rightarrow \boxed{m\ddot{x} + k_e x = 0}$$

(e). Calculate the bracket's maximum deflection (i.e., amplitude of vibration), in meters.

Sol$^n$: $x = C_1 \cos(\omega t) + C_2 \sin(\omega t)$

$\therefore \dot{x} = -C_1 \omega \sin(\omega t) + C_2 \omega \cos(\omega t)$

ICs: $x_0 = 0$

$x(t=0) = C_1 + 0 = 0 \Rightarrow \boxed{C_1 = 0}$

$\dot{x}_0 = I/m$

$\dot{x}(t=0) = 0 + C_2 \cdot \omega = \frac{I}{m} \Rightarrow \boxed{C_2 = \frac{I}{m \cdot \omega}}$

$I = \int F\,dt = (m V_0)_{bird} = 1\,kg \times 72\,\frac{km}{hr} \cdot 1000\,\frac{m}{km} \cdot \frac{1\,hr}{3600\,s}$

$= 20$ kg m/s

$\omega = \sqrt{\frac{k_e}{m}} = \sqrt{\frac{17070}{3}} = 75.43$ rad/s

$\therefore C_2 = \frac{20}{3 \times 75.43} = \boxed{0.088\ m}$

$\therefore$ Response is $x(t) = 0.088 \sin(75.43\,t)$

where amplitude $(X) = \boxed{0.088\ m}$ ans.