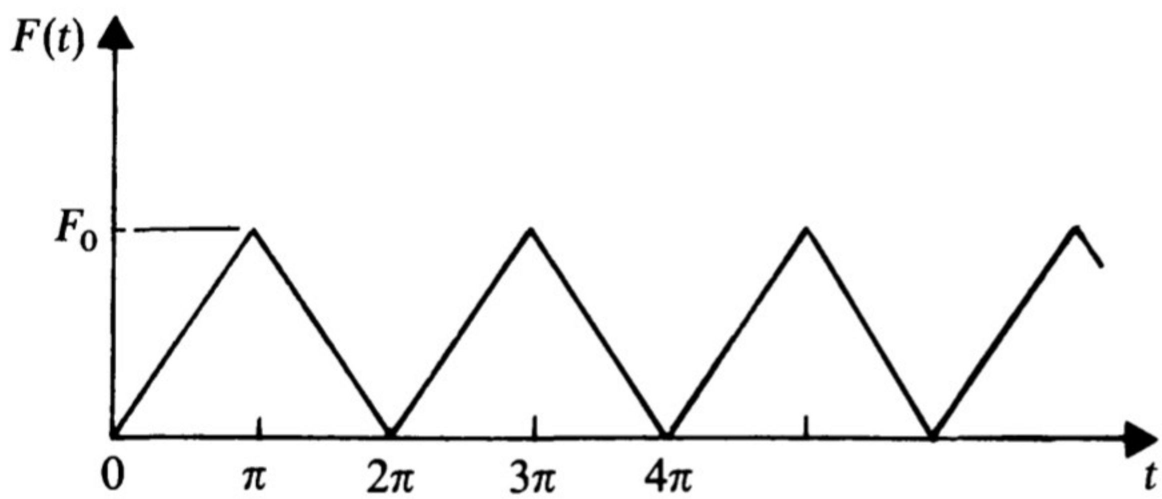
Practice Problem Set 5B

Vibration Response to Periodic Forcing Function using Fourier Series Approximation

Do the following problems from the book:

5.7: Find the vibration response, $x(t)$, of an <u>undamped</u> 1-DOF system subject to the periodic forcing function below.

5.8: Find the vibration response, $x(t)$, of a <u>damped</u> 1-DOF system subject to the periodic forcing function below.



Also, why don't you try plotting the solutions?

SCROLL

DOWN

FOR

SOLUTION

(But don't get tempted by the dark side. Resist! Use the, um, Force?)

ARE

YOU

SURE?

(Go back up and think harder? Also, what exactly are you looking for in the solution below?)

SOLUTION

5.7

(7) $m\ddot{x} + kx = F$

$$F(t) = \frac{F_0}{2} - \sum_{n=1,3,5}^{\infty} \frac{4F_0}{(n\pi)^2}\cos nt$$

$$= \frac{F_0}{2} - \sum_{n=1,3,5}^{\infty} \frac{4F_0}{(n\pi)^2}\sin\left(nt+\frac{\pi}{2}\right)$$

$$\omega = \sqrt{\frac{k}{m}} \quad , \quad \omega_f = 1$$

$$x_h = A\sin(\omega t + \phi)$$

$$x_p = \frac{F_0}{2k} + \sum_{n=1,3,5}^{\infty} \frac{F_n/k}{(1-r_n^2)}\sin(\omega_n t + \phi_n - \psi_n)$$

$$\omega_n = n\,\omega_f = n \ , \quad r_n = \frac{\omega_n}{\omega} = \frac{n}{\omega}$$

$$\phi_n = \frac{\pi}{2} \ , \quad \psi_n = \tan^{-1}(0) = 0$$

$$F_n = \begin{cases} 0 & \text{if } n \text{ is even} \\ \dfrac{-4F_0}{(n\pi)^2} & \text{if } n \text{ is odd} \end{cases}$$

$$x = A\sin(\omega t + \phi) + \frac{F_0}{2k} - \sum_{n=1,3,5}^{\infty} \frac{4F_0\,\omega^2}{(n\pi)^2 k(\omega^2 - n^2)}\sin\left(nt + \frac{\pi}{2}\right)$$

5.8

(8) $$m\ddot{x} + c\dot{x} + kx = F$$

$$F(t) = \frac{F_0}{2} + \sum_{n=1,3,5}^{\infty} \frac{(-4F_0)}{(n\pi)^2} \sin\left(nt + \frac{\pi}{2}\right)$$

$$\omega = \sqrt{\frac{k}{m}} \quad , \quad \xi = \frac{c}{2m\omega}$$

$$\omega_d = \omega\sqrt{1-\xi^2}$$

$$\therefore \; x = x_h + x_p$$

$$= A e^{-\xi\omega t} \sin(\omega_d t + \phi) + \frac{F_0}{2k}$$

$$+ \sum_{n=1,3,5}^{\infty} \frac{F_n / k}{\sqrt{(1-r_n^2)^2 + (2r_n\xi)^2}} \sin(\omega_n t + \phi_n - \psi_n)$$

where

$$\omega_f = 1 \quad , \quad \omega_n = n\,\omega_f = n$$

$$F_n = \begin{cases} 0 & \text{if } n \text{ is even} \\ \dfrac{-4F_0}{(n\pi)^2} & \text{if } n \text{ is odd} \end{cases}$$

$$r_n = \frac{\omega_n}{\omega} = \frac{n}{\omega} \quad , \quad \phi_n = \frac{\pi}{2}$$

$$\psi_n = \tan^{-1}\left(\frac{2r_n\xi}{1-r_n^2}\right) = \tan^{-1}\left(\frac{nc\omega^2}{m\omega^2(\omega^2-n^2)}\right)$$