Feb 10, 2021

Solve: $\ddot{x} + 16x = 10 + 3e^{t}$

ICs: $\begin{cases} x_0 = 10 \\ \dot{x}_0 = 0 \end{cases}$

by hand

(MUC) → x_p

↓

plot

10 min.

code

Sol: $x = x_h + x_p$

$\boxed{x_h}$ $\quad \ddot{x} + 16x = \cancel{10 + 3e^{t}}\ \emptyset$

Char. Eq: $p^2 + 16 = \emptyset$

$\Rightarrow p = \emptyset \pm 4i$

α β

$p = \alpha \pm i\beta$

$\therefore x_h = X \cdot e^{\alpha t} \sin(\beta t + \phi)$

1

$x_h = X \sin(4t + \phi)$

$\boxed{x_p}$ M.v.u.C.

① $f = 10 + 3e^t$

$f' = 3e^t$

$f'' = 3e^t$

② e^t, const.

③ let $x_p = C_1 + C_2 e^t$

④ $\dot{x}_p = C_2 e^t$

$\ddot{x}_p = C_2 e^t$

⑤ bring into EOM

$(\ddot{x}_p + 16 x_p = 10 + 3e^t)$

$\Rightarrow C_2 e^t + 16 \cdot (C_1 + C_2 \cdot e^t)$
$= 10 + 3e^t$

$\Rightarrow 16 C_1 + 17 C_2 \cdot e^t = 10 + 3e^t$

Equate both sides :

$\boxed{\text{Const.}}$ $16 C_1 = 10 \Rightarrow C_1 = \frac{5}{8}$

$\boxed{e^t}$ $17 C_2 = 3 \Rightarrow C_2 = \frac{3}{17}$

⑥ $x_p = \frac{5}{8} + \frac{3}{17} e^t$

$\therefore$ sol:

$$\boxed{x = X \cdot \sin(4t + \phi) + \frac{3}{17}e^{t} + \frac{5}{8}} \quad \text{— (1)}$$

ICs $\begin{cases} x_0 = 10 \\ \dot{x}_0 = 0 \end{cases}$

$$\dot{x} = 4X\cos(4t + \phi) + \frac{3}{17}e^{t} \quad \text{— (2)}$$

Apply ICs:

Eq. (1): $x(t=0) = 10 = X\sin\phi + \frac{3}{17} + \frac{5}{8}$ — (3)

Eq. (2): $\dot{x}(t=0) = 0 = 4X\cos\phi + \frac{3}{17}$ — (4)

Solving (3), (4) simultaneously:

Eq. (3): $X \sin\phi = 10 - \frac{3}{17} - \frac{5}{8}$ (5)

Eq. (4): $X \cos\phi = -\frac{3}{68}$ (6)

$\frac{(5)}{(6)}$: $\tan\phi = -\frac{10 - \frac{3}{17} - \frac{5}{8}}{3} \cdot 68$

$\Rightarrow \phi = \tan^{-1}\left(\downarrow \right)$

$\approx \boxed{-1.566 \text{ rad}}$

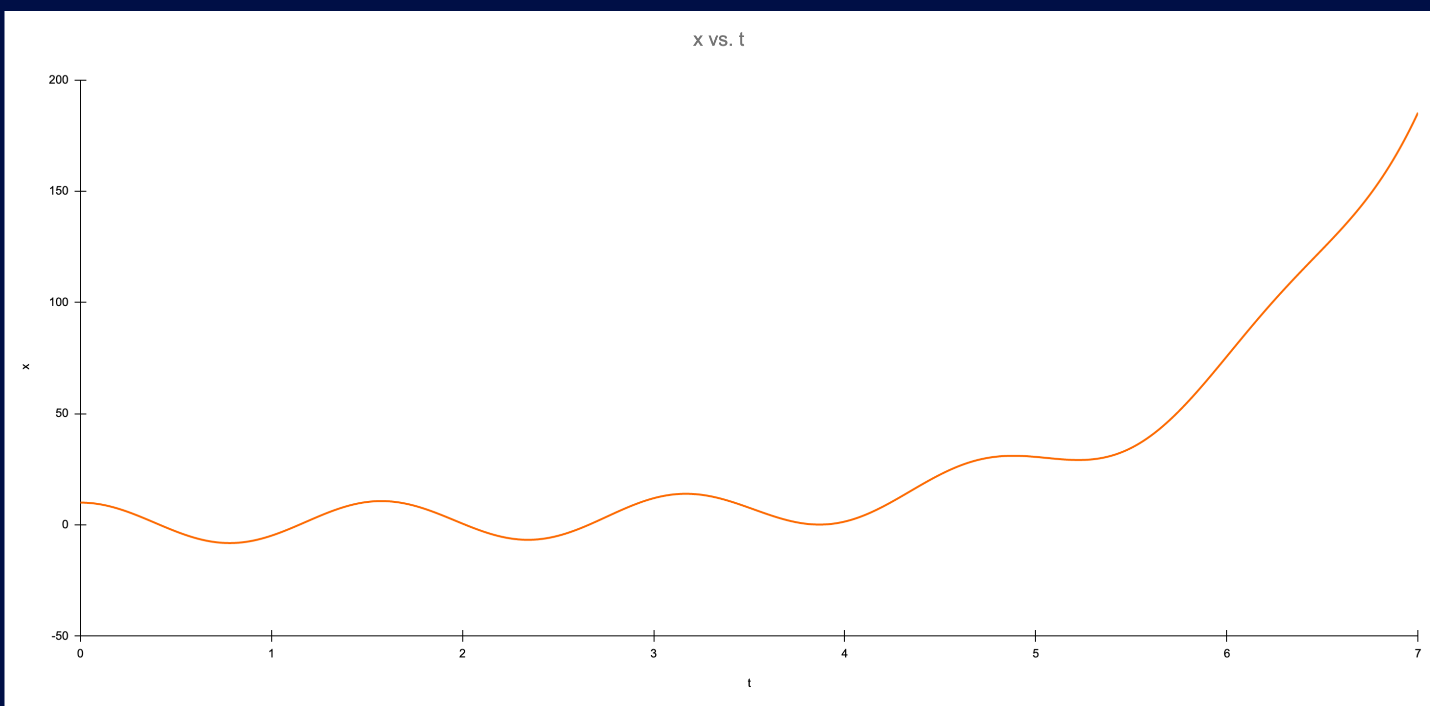
(6): $X = -\frac{3/68}{\cos(-1.566)}$

$\approx \boxed{-9.2}$

$\therefore$ complete sol:

$$\boxed{x(t) = -9.2\sin(4t - 1.566) + \frac{3}{17}e^{t} + \frac{5}{8}}$$

plot this in spreadsheet



Now, solving the same ODE in Matlab: